QF3101 Assignment Report

Low Jia Jun

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1 3-month SOFR Futures and Deferred FRA.

1.1 Stage 1: Data collection and preparation.

As stated in the assignment requirements, the choice of business day is **27 March 2024**, which is stated in Cell A1 in QF3101_FinalSpreadsheet.xlsm. For my choice of business day, I have collected the settlement prices of SR3 futures contracts with the name Jun 2024, Sep 2024, ..., Sep 2033 from the CME page under the column "Prior Settle" in the Quotes sub page (Figure 1[1]) in Cell B4:B41 and labeled this column as "Prior Settle" in Cell B3.

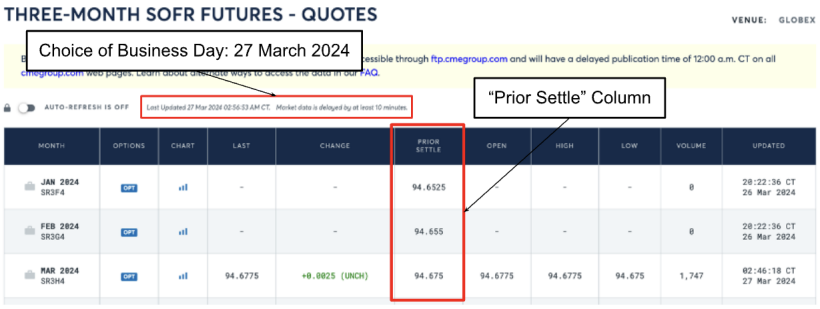


Figure 1: Screenshot of CME page of futures prices.

As stated in the assignment requirements, for each futures price P, I have calculated the corresponding 3-month SOFR by $(100 - P)\%$ to be stored in Cell C4:C41 labelled as "SR3Rates" in Cell C3. For instance, for the Jun 2024 contract, $P = 94.885$, hence the corresponding 3-month SOFR is calculated by $(100 - 94.885)\% = 5.115\%$. This is done for Jun 2024, Sep 2024, ..., Sep 2033 contracts using cell formula $(100 - B\#)\%$, where $B\#$ is the futures price quoted for the relevant contracts. In the preceding example, $B\# = B4$.

[1] I have saved the webpage under QF3101_Figure1.pdf for full reference if required.

As stated in the assignment requirements, I have obtained the settlement dates of the SR3 futures mentioned (i.e. for Jun 2024, Sep 2024, ..., Sep 2033) from the CME page to be stored in Cell D4:D41 in DD-MM-YYYY format labelled as "Settlement" in Cell D3.

As stated in the assignment requirements, to compute the number of days for each contract quarter, take the settlement date of that particular contract quarter and minus it with the settlement date of the previous contract quarter. For instance, to obtain the number of days for Sep 2024 futures, we take the settlement date of Sep 2024 futures contract and minus it with the settlement date of Jun 2024 futures contract. This is done by using the cell formula $D5 - D4$. The same is repeated for Dec 2024, ..., Sep 2033 futures using cell formula $D6 - D5, ..., D41 - D40$ respectively. The result is stored in Cell E5:D41 labelled as "SR3DaysInQuarter" in Cell E3.

1.2 Stage 2: Implied Forward SOFR term structure.

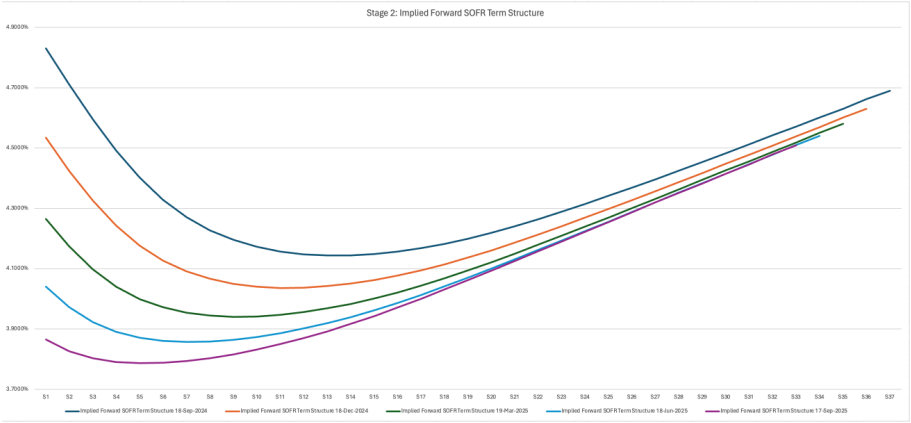


Figure 2: Term Structure obtained for the reference dates of 18-Sep-2024, 18-Dec-2024, 19-Mar-2025, 18-Jun-2025 and 17-Sep-2025.

A VBA **cell-array** function $GetTermStructure()$[2] was called **five** times to obtain the following **five** term structures presented in Figure 2. For the implied forward SOFR term structure for the reference date of 18 September 2024, we observe an initial rate drop from around S1 to S7, the curve then slopes gradually upward from around S7 to S37. A similar pattern follows for the other implied forward SOFR term structure for the reference dates of 18 December 2024, 19 March 2025, 18 June 2025 and 17 September 2025,

[2]For example, for the implied forward SOFR for the reference date of 18 September 2024, we pass in SR3Rates from Cell C5:C41 and SR3DaysInQuarter from Cell E5:E41 to obtain the dark blue curve in Figure 2.

converging to around 4.6%. This trend is consistent across the various reference dates, indicating expectations of rising three-months SOFR rates over time.

1.3 Stage 3: Pricing Deferred Forward Rate Agreement.

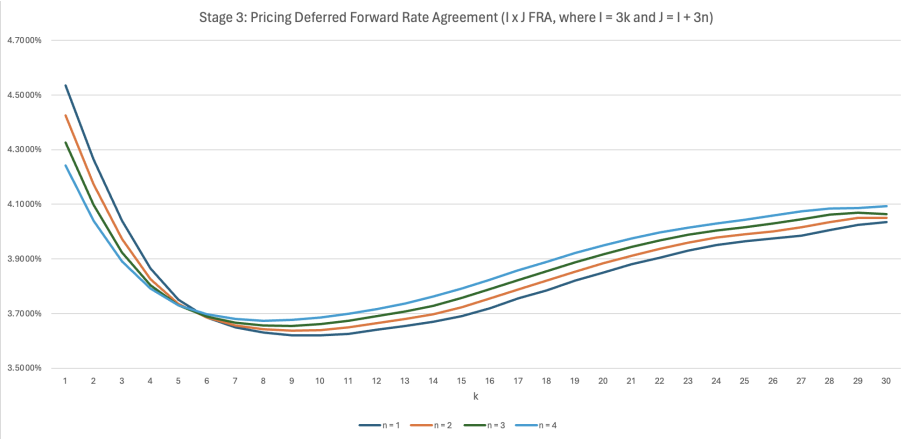


Figure 3: FRA rates obtained for deferred $I \times J$ FRA with starting date being 18-Sep-2024 for $I \times J$ FRA where $J = I + 3n$ for $n = 1, 2, 3, 4$ and $I = 3k$ for $k = 1, 2, ..., 30$.

A VBA **user-defined** function $GetFRARate()$[3] was called **120** times to obtain the following **four** price trends for the **four** different reference rates used, the 3-month SOFR in the graph of $n = 1$, 6-month SOFR in the graph of $n = 2$, 9-month SOFR in the graph of $n = 3$ and 12-month SOFR in the graph of $n = 4$ when the FRA contract ends at time $I = 3k$ where $k = 1, 2, ..., 30$.

In general, regardless of the **four** different reference rates used, there is an increasing trend for FRA which expires at time $I = 3k$, for $k \geq 7$. The price of FRA follows a concave upward and increasing trend from $k \geq 7$ till around $17 \leq k \leq 20$, which then after follows concave downward and increasing trend, converging to around 4.06% at $k = 30$. However, for a FRA which expires at time $I = 3k$, for $1 \leq k < 7$, there is a concave upward and decreasing trend. This suggests a market expectation of rising three-month SOFR rates in the near term, followed by a stablisation or slight decrease in the long term.

[3]For example, for the deferred $I \times J$ FRA where $J = I + 3n$, $n = 1$ and $I = 3k$ for $k = 1$ which imply that we pass in I = 3 in Cell P4, J = 6 in Cell Q4, the implied forward term structure with reference date on 18 Sep 2024 from Cell I5:I41 and the number of days for rates in the term structure from Cell F5:F41. This is done repeatedly for other values of $k = 2, 3, ..., 30$ to obtain the dark blue curve in Figure 3.

2 Pricing an Interest Rate Swap and Tracking its Value.

2.1 Stage 1: Data collection and preparation.

As stated in the assignment requirements, I have collected the USD LIBORs for "first" business day of the month in the United States in 2010 for D where D is given by {January 04 2010, Feruary 01 2010, ..., December 01 2010} from the Global-Rates page, where I selected the required maturities from 1 month to 12 months for each D. D is stored in Cell A2:A13 and labeled as "D" in Cell A1, while the rates for each maturities from 1 month to 12 months for each D is stored in Cell B2:M13 labelled as "1", "2", ..., "12" in Cell B1, C1, ..., M1 respectively.

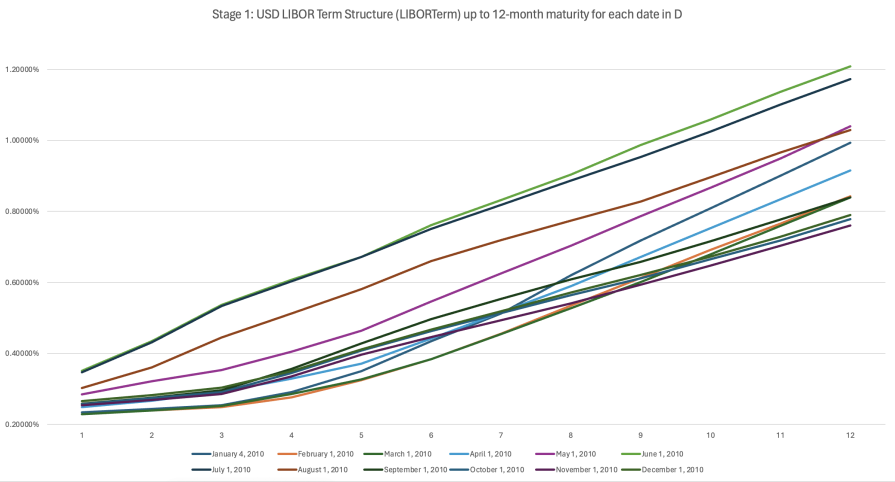


Figure 4: USD LIBOR term structure obtained up to 12-month maturity for each date in D

As stated in the assignment requirements, I have obtained the 12 term structures obtained and plotted it in Figure 4.

In general, for every "first" business day of the month in the United States in 2010, there is an increasing trend in USD LIBOR for maturities increasing from 1 month to 12 months. This suggests a market expectation of rising LIBOR rates over the year.

In order to obtain the number of days for each maturity from 1 month to 12 months for the term structure obtained for each date in D, a larger set of dates E was created to have the dates {January 03 2011, February 01 2011, ... December 02 2011} added to D. E is stored in Cell A16:A29 and labeled as

"E" in Cell A15. For instance, for the term structure obtained for **January 04 2010**, the difference between the two dates January 03 2010 and February 01 2010 gives the number of days for the 1 month maturity, this is calculated using cell formula A17-A16 and stored in cell B16. The number of days for the 2 months maturity, up to the 12 months maturity for the term structure obtained for **January 04 2010** is calculated with a similar cell formula, with the difference in cells being k where k represents the k-th month maturity for $k = 1, 2, ..., 12$. For the term structure obtained for **January 04 2010**, the k-month maturity is stored in Cell B16:M16. The above process is repeated for the term structure obtained for **February 01 2010, ..., December 01 2010** and their corresponding k-month maturity is stored in Cell B17:M17.

2.2 Stage 2: Swap Rate of a one-year Interest Rate Swap.

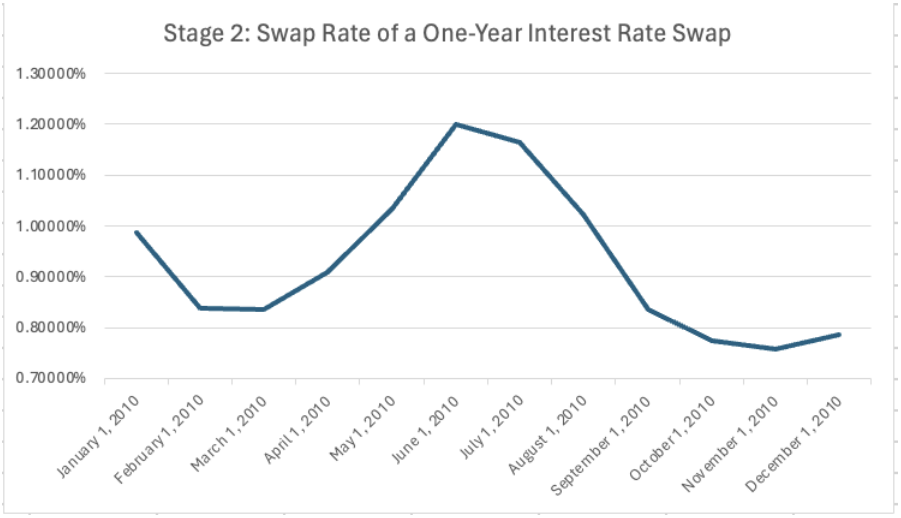


Figure 5: Swap Rates obtained for each of the one-year Interest Rate Swap starting on the dates given in D.

A VBA **user-defined** function $GetSwapRate()$[4] was called **12** times to obtain the following **12** Swap Rates of a One-Year Interest Rate Swap starting from dates given in D presented in Figure 5.

In general, for the swap rates obtained for each of the one-year Interest Rate Swap starting on the dates given in D do not correspond to any standard

[4]For example, for swap rate obtained for each of the one-year interest rate swap starting from January 4 2010, we pass in LIBOR term structure for January 4 2010 from Cell B2:M2 and the corresponding number of days for rates in this term structure from Cell B16:M16 to obtain the first data point near January 1 2010 in Figure 5.

mathematical function (i.e., concave up or concave down, increasing or decreasing). However, there is an initial dip of Swap Rate of One-Year Interest Rate Swap starting from D = January 01 2010 to D = March 01 2010, followed by a sharp rise from D = March 01 2010 to D = July 01 2010 before declining again from D = July 01 2010 to December 01 2010. This observation could be attributed to market volatility and changes in LIBOR rate expectations during the period.

2.3 Stage 3: Tracking Swap Value of a one-year Interest Rate Swap.

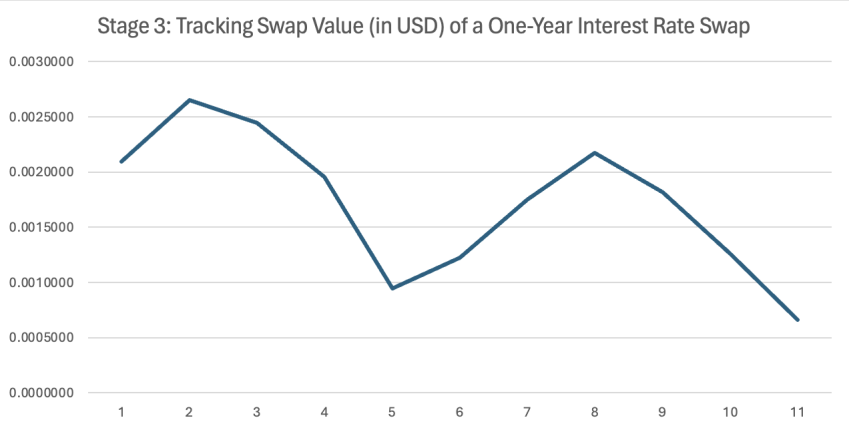


Figure 6: Swap Value (in USD) of a One-Year Interest Rate Swap on the first business day of the k-th month after inception for $k = 1, ..., 11$.

A VBA **user-defined function** $GetSwapValue()$[5] was called **11** times to obtain the following **11** Swap Values (in USD) of a One-Year Interest Rate Swap starting on the first business day of the k-th month after inception for the one-year interest rate swap beginning on **January 04 2010**.

In general, there is a general decline in value from $k = 2$ to $k = 11$ month after inception with the exception of an increase in value from $k = 5$ to $k = 8$ month after inception before following the general decline in value from $k = 8$ to $k = 12$ once again. This pattern suggests that the value of the swaps was influenced by changing LIBOR market conditions and LIBOR rate expectations.

[5]For example, we obtain the swap value (in USD) of a one-year interest rate swap beginning on the first business day of the $k = 1$ month after 04 January 2010 (inception) by passing January 4 2010 swap rate in Cell P16, $k = 1$ in Cell O30, February (1 month after inception) term structure in Cell B3:M3 and its corresponding number of days in the February term structure in Cell B17:M17 to obtain the first data point at $k = 1$ in Figure 6.